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A Note on Noncommutative Chern-Simons Theories

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Abstract

The three dimensional Chern-Simons theory on $\mathbb{R}^2_\theta \times \mathbb{R}$ is studied. Considering the gauge transformations under the group elements which are going to one at infinity, we show that under arbitrary (finite) gauge transformations action changes with an integer multiple of 2π *if*, the level of noncommutative Chern-Simons is *quantized*. We also briefly discuss the case of the noncommutative torus and some other possible extensions.

1 Introduction

The Chern-Simons theories have been considered in mathematics and physics literature extensively. From the physics side they shed light on the planar physics -physics in two spatial directions- and in particular quantum Hall effect and superconductivity, for a review see e.g. [1, 2]. From the mathematical point of view, being a topological theory, it has been used to give a "physical" interpretation to topological invariants of knot theory and Jones polynomials [3, 4].

The pure Chern-Simons theory is described by the action:

$$S_{\rm CS} = \frac{1}{4\pi\nu} \int_M \epsilon^{\mu\nu\alpha} {\rm Tr} \left(A_\mu \partial_\nu A_\alpha + \frac{2i}{3} A_\mu A_\nu A_\alpha \right) \ , \tag{1.1}$$

where M is (an oriented) 3-manifold and A_μ are the connections corresponding to a simple compact gauge group G. As we can see from the action the metric of the manifold M is not appearing in (1.1). The factor $\frac{1}{\nu}$ (the Chern-Simons coupling) is usually called the "level" of the Chern-Simons theory and the gauge invariance of the action under finite gauge transformations implies that is should be quantized, i.e. $\frac{1}{\nu} \in \mathbb{Z}$ [5] which is a reflection of a general mathematical fact that the group of continuous maps $M \to G$ is not connected. In the homotopical classification of such maps one meets the fact that $\Pi_3(G) = \mathbb{Z}$ for any simple compact group G.

As a field theory one can study action (1.1). The classical equations of motion are $F_{\mu\nu} = \partial_{[\nu} A_{\mu]} - i A_{[\mu} A_{\nu]} = 0$, the classical paths are flat connections. In fact it turns out that the partition function of this theory can be expressed in terms of moduli parameters of the flat connections [3].

The Yang-Mills-Chern-Simons theory is obtained by adding the usual Yang-Mills action to $S_{\rm CS}$:

$$S = \frac{4\pi}{g^2} \int_M {\rm Tr} \left(F_{\mu\nu} F^{\mu\nu} \right) + \frac{1}{4\pi\nu} \int_M \epsilon^{\mu\nu\alpha} {\rm Tr} \left(A_\mu \partial_\nu A_\alpha + \frac{2i}{3} A_\mu A_\nu A_\alpha \right) \ ,$$

where $F_{\mu\nu}$ is the field strength. The pure Chern-Simons theory is then recovered at the strong coupling limit of the Yang-Mills part ($g \to \infty$) while keeping the ν (or the Chern-Simons coupling) fixed. The Chern-Simons term in the Yang-Mills-Chern-Simons theory can be interpreted as a (topological) mass term for the Yang-Mills part with effective mass $m = \frac{1}{(2\pi g)^2 \nu}$ [5]. However, in order to perform the usual field theory calculations one needs to fix the gauge symmetry. Although the gauge fixing terms do depend on the metric chosen on M, the full quantum theory (the partition function) remains topological [3, 6, 7].

Perturbative loop calculations have been performed for both pure Chern-Simons [8] and for Yang-Mills-Chern-Simons [9]. Their results indicate that the Chern-Simons level for $SU(N)$ gauge theory is renormalized as:

$$\frac{1}{\nu_{\rm ren}} = \frac{1}{\nu_{\rm class}} + N \ . \tag{1.2}$$

Moreover it has been shown that this one loop result does not receive any further corrections from the higher loops [10, 11]. In fact this exact result has been proved using the BRST (ward identity) and also the vector supersymmetry of Chern-Simons theory [11]. This is remarkable because it assures the level quantization to all loops order.

The field theories on the noncommutative spaces (in particular noncommutative Moyal plane and noncommutative tori) has been considered a few years ago, e.g. see [12]. However, before a new re-motivation from string theory [13], it was not studied extensively. Generally the noncommutative version of a given field theory is obtained by replacing the usual product of the functions (fields) by the Moyal $\star$-product, namely

$$\begin{aligned} f(x) \star g(x) &= \exp(\frac{i}{2}\theta^{\mu\nu}\frac{\partial}{\partial\xi^\mu}\frac{\partial}{\partial\zeta^\nu}) \ f(x+\xi)g(x+\zeta) \ |_{\xi=\zeta=0} \\ &= f(x) \ \exp(\frac{i}{2}\theta^{\mu\nu}\overleftarrow{\partial_\mu} \ \overrightarrow{\partial_\nu})g(x) \ , \end{aligned} \tag{1.3}$$

where $\theta^{\mu\nu}$ is a constant anti-symmetric tensor. The Moyal bracket is then defined as

$$\{f,g\} = f \star g - g \star f \ . \tag{1.4}$$

It is easily seen that $\{x^\mu, x^\nu\} = i\theta^{\mu\nu}$. For a review on noncommutative field theories and some helpful identities on $\star$-product see [14]. In particular one can construct noncommutative version of the $U(N)$ gauge theories, which give rise to the noncommutative deformed version of the usual gauge symmetry [13]. We should remember that so far only the noncommutative $U(N)$ theory have been considered perturbatively in the existing literature.

The noncommutative extension of the Chern-Simons (NCCS) theories have also been considered [15, 16, 17, 18, 19, 20]. Physically the noncommutative ($U(1)$) Chern-Simons theory seems to serve a natural description for the Wigner crystal-quantum Hall fluid phase transition [21]. In the context of quantum Hall effect, the parameter ν (inverse of the "Chern-Simons level") appears to be the *filling fraction* for quantum Hall states. Performing the quantization in the matrix (discrete) description of noncommutative ($U(1)$) Chern-Simons , it has been shown that the factor ν is basically related to the statistics of the particles described by NCCS action. More precisely upon exchange of any two particles in the corresponding n

particle state, we obtain a factor of $\exp(\frac{i\pi}{\nu})$:

$$\Psi(1,2,...,i,...,j,...,n) = \exp\left(\frac{i\pi}{\nu}\right)\ \Psi(1,2,...,j,...,i,...,n)\ , \tag{1.5}$$

where $\Psi(1,2,..,i,...,j,...,n)$ is an n particle state. Therefore the NCCS particles are in general anyonic, if ν^{-1} is an arbitrary real number [21]. We note that all the noncommutative Chern-Simons theories with their coupling $\frac{1}{\nu}$ differing by an *even* integer, describe the same statistics.

In this work, we study the behaviour of the ($U(1)$ and $U(N)$) Chern-Simons theories on $\mathbb{R}^2_\theta \times \mathbb{R}$, $\mathbb{R}^2_\theta$ being the noncommutative Moyal plane, under both infinitesimal and finite gauge transformations. We show that the partition function (quantum theory) is invariant under the gauge transformations, provided that $\frac{1}{\nu}$ is an integer. In the usual Chern-Simons terminology this means that we again face the level quantization. From the noncommutative Chern-Simons particles point of view, this means that NCCS describes only fermions or bosons, and not anyons.

The paper is organized as follows. In section 2, we briefly review the results of [17, 18] on the perturbative analysis of NCCS theories. In section 3, which contains our main result, we study the invariance of NCCS action for arbitrary (finite) gauge transformations and show that the full quantum action is invariant, if the level is quantized. The last section is devoted to discussions and open questions.

2 Noncommutative Chern-Simons theories: perturbative analysis

In this section we review the results of loop calculations for noncommutative Chern-Simons theories [17, 18]. The action for the noncommutative extension of Chern-Simons theory is

$$S_{\rm NCCS} = \frac{1}{4\pi\nu}\int_M \epsilon^{\mu\nu\alpha}{\rm Tr}\left(A_\mu \star \partial_\nu A_\alpha + \frac{2i}{3}A_\mu \star A_\nu \star A_\alpha\right)\ , \tag{2.1}$$

where $\star$-product is defined in (1.3), A_μ take values in $U(N)$ algebra (unitary $N\times N$ matrices) and Tr is basically trace over the $U(N)$ indices. Here we consider the base manifold M which can be separated into a two dimensional (noncommutative) subspace times $\mathbb{R}$ or S^1 and further we assume that we can define the $\star$-product introduced earlier (1.3), i.e. we consider the noncommutative plane, noncommutative cylinder [22, 23] and noncommutative torus.

However, we should remind ourselves that for the case of torus, depending on the fact that the noncommutativity per unit volume is rational or irrational, one can realize two completely different cases. For the rational noncommutative torus, our gauge theory (noncommutative Chern-Simons) can be mapped into a commutative theory with a non-trivial background flux while it is not possible for the irrational noncommutative torus [24]. Here we do not consider the torus case in detail and will only make some remarks passing at the end of section 3 and 4.

The action (2.1) is invariant under infinitesimal gauge transformations:

$$A_\mu \longrightarrow A'_\mu = A_\mu + \partial_\mu \lambda + i(\lambda \star A_\mu - A_\mu \star \lambda) \ , \qquad \lambda \in U(N) \ . \tag{2.2}$$

However, the case of finite gauge transformations should be studied separately and this we will do in the next section.

Analogous to the commutative case, one can define the noncommutative Yang-Mills-Chern-Simons theory by adding the noncommutative Yang-Mills action to (2.1) [18]. The perturbative loop calculations have also been done for both pure NCCS [17] and for noncommutative Yang-Mills-Chern-Simons [18]. The pure Chern-Simons action at one loop level can once again be recovered by sending the gauge coupling to infinity.

The key observation in performing the noncommutative loop calculations in general, is that the $\star$-product (and hence the Moyal bracket) in the Fourier modes take a simple form, namely

$$\{\mathrm{e}^{ik\cdot x}, \mathrm{e}^{ip\cdot x}\} = 2i\sin(\frac{1}{2}k\theta p)\mathrm{e}^{i(k+p)\cdot x} \ , \qquad k\theta p = k_\mu \theta^{\mu\nu} p_\nu \ ,$$

and therefore the loop integrals in general can be decomposed in two parts, one which contains the θ factor ($\mathrm{e}^{ik\theta p}$), the non-planar part, and one which do not depend on θ, the planar part [25, 26]. At one loop level the non-planar parts are (UV) finite while they show an IR divergence. This is known as IR/UV mixing which is a peculiar feature of noncommutative field theories [25, 26].

For the gauge theories however, the divergent part of the loop integrals, compared to the commutative counter-part, contains an extra factor of $4\sin^2(\frac{1}{2}k\theta p) = 2(1+\cos(k\theta p))$. The loop integrals lead to the usual (commutative) β-function equation but the $C_2(G)$ (quadratic Casimir of the gauge group) is now replaced by $2 \times C_2(G)$ [27] which for NC$U(N)$ that is $2N$.

For the noncommutative Chern-Simons theory (*as a limit of* noncommutative Yang-Mills-

Chern-Simons) the loop calculations have been performed and shown that [18] *

$$\frac{1}{\nu_{\rm ren}} = \frac{1}{\nu_{\rm class}} + 2 \times N \ . \tag{2.3}$$

The novel point shown explicitly in [18] is that all the non-planar diagrams vanish in the pure Chern-Simons limit, i.e. we are *not* going to find any further IR divergence due to these terms. However, if we just start with the pure Chern-Simons we will persist with the IR divergences coming from non-planar diagrams [17]. This can be understood by noting that the Chern-Simons term introduces a topological mass term in the Yang-Mills action [5], and this mass term serves as a natural IR regulator, which, even in the pure Chern-Simons limit, tames the IR divergences of non-planar diagrams. Presumably the eq.(2.3), although being a one loop result, holds for all loop orders, similar to the commutative case [11]. In fact this is expected because we can again recognize the BRST and vector supersymmetry in this case [17, 28].

The eq.(2.3) is also remarkable noting the statistics of the noncommutative Chern-Simons particles [21]: the quantum corrections to $\frac{1}{\nu}$ are not going to change the statistics of these particles, this is in fact what is expected from a well-defined quantum theory.

The other important property of the action (2.1) is that it is independent of the choice of metric on M (note that metric is not involved in $\star$-product). However, in order to be treated as a topological field theory one should guarantee that this property remains at the level of the quantum partition function. On the other hand to perform the path integral we should fix the gauge. Adding the gauge fixing and ghost terms to the action spoils the metric independence, and hence the topological nature of the theory becomes questionable [6]. In the usual commutative Chern-Simons theories this is overcome recalling the BRST symmetry of the full gauge fixed action and the fact that the gauge fixing and the ghost part of the action can be expressed as commutator of BRST charge with a local function of the fields [6]. This argument also holds for the noncommutative case. The existence of the BRST symmetry have been shown in [17]. Also there it has been shown that the metric dependent (gauge fixing) part of the action is a BRST exact form †. This guarantees the

*Actually the results of [18] differ with ours in the factor of 2. That is related to the *improper normalization* they use.

†Besides the action, in the path integral there is also the measure, which is the same for a commutative theory and its noncommutative counter-part [14]. However, as has been discussed in [7] the measure is metric independent.

metric independence of the noncommutative Chern-Simons at quantum level. Hence, the noncommutative Chern-Simons can be treated as a topological field theory.

3 The invariance of noncommutative Chern-Simons under finite gauge transformations

In this section we would like to address another feature of noncommutative Chern-Simons theories on the $\mathbb{R}^2_\theta \times \mathbb{R}$. As we have discussed the noncommutative Chern-Simons action (2.1) is invariant under small gauge transformations. However, it is also important to study its behaviour under finite noncommutative gauge transformations defined by

$$A_\mu \longrightarrow A^g_\mu = g^{-1} \star A_\mu \star g + i g^{-1} \star \partial_\mu g \ , \tag{3.1}$$

where g is valued in NC$U(N)$ group: g is an $N \times N$ matrix satisfying

$$g^\dagger \star g = g \star g^\dagger = \mathbf{1} \ . \tag{3.2}$$

One can show that

$$\begin{aligned} g &= (\mathrm{e}\star)^{i\lambda} \equiv 1 + i\lambda - \frac{1}{2}\lambda \star \lambda - \frac{i}{3!}\lambda \star \lambda \star \lambda + \cdots , \\ g^\dagger &= g^{-1} = (\mathrm{e}\star)^{-i\lambda} \ , \end{aligned} \tag{3.3}$$

with λ being a $N \times N$ matrix, solves (3.2). We recall that, according to definition of $NCU(N)$ algebras $g^{-1}dg$ should be a member of the algebra of the compact operators on $\mathbb{R}^2_\theta \times \mathbb{R}$ [29]. In addition, we also impose the boundary condition

$$g_{x \to \infty} \to \mathbf{1}. \tag{3.4}$$

Under the above gauge transformations the action (2.1) transforms as $S \to S + \delta_g S$, where

$$4\pi\nu \ \delta_g S = -\int \epsilon^{\mu\nu\alpha} \Big[\partial_\mu \mathrm{Tr}(\partial_\nu g \star g^{-1} \star A_\alpha) - \frac{i}{3} \mathrm{Tr}(g^{-1} \star \partial_\mu g \star g^{-1} \star \partial_\nu g \star g^{-1} \star \partial_\alpha g) \Big]. \tag{3.5}$$

The first term in the above is a total derivative and vanishes due to boundary conditions, while the second term is a non-trivial one, usually called the "winding number" of the group. For the usual compact groups, this winding number is an integer. However, for the noncommutative case one should redo the corresponding computations, and this is what we

do in the following. In order to show how the method works, let us first consider the NC$U(1)$ case, and then we will generalize it to NC$U(N)$.

The NC$U(1)$ winding number

In this case g is just a function (without matrix indices), but we should remember that in general it is θ dependent. First taking derivatives of (3.2) we find two very useful identities:

$$\partial_\mu g^{-1} \star g = -g^{-1} \star \partial_\mu g \ , \tag{3.6}$$

and

$$\frac{-i}{4}\partial_{[\mu} g^{-1} \star \partial_{\nu]} g = \frac{\delta}{\delta\theta^{\mu\nu}} g^{-1} \star g + g^{-1} \star \frac{\delta}{\delta\theta^{\mu\nu}} g \ . \tag{3.7}$$

The identity (3.7) is obtained by taking the derivative of (3.2) with respect to $\theta_{\mu\nu}$. Using the identity (3.6) we find

$$g^{-1} \star \partial_\mu g \star g^{-1} \star \partial_\nu g = -\partial_\mu g^{-1} \star \partial_\nu g \ .$$

Then we use the identity (3.7), and again (3.6). Recalling the boundary condition (3.4) we can drop the total derivative term and we remain with

$$\int \epsilon^{\mu\nu\alpha} \left(g^{-1} \star \partial_\mu g \star g^{-1} \star \partial_\nu g \star g^{-1} \star \partial_\alpha g \right) = 4i \int \epsilon^{\mu\nu\alpha} \ \frac{\delta}{\delta\theta^{\mu\nu}} \left(g^{-1} \star \partial_\alpha g \right) \ .$$

If we parameterize the $\mathbb{R}$ part by t and the $\mathbb{R}^2_\theta$ by x, y, we have

$$\delta_g S = \frac{-i}{2\pi\nu} \epsilon^{ij} \frac{\delta}{\delta\theta^{ij}} \int dxdy \int dt \left(g^{-1} \partial_t g \right) \ , \tag{3.8}$$

where $i, j = 1, 2$. Now, noting the representation of g, (3.3), one can prove that

$$g^{-1} \star \partial_\alpha g = i\partial_\alpha \lambda + \{B_\alpha, \lambda\} \ , \tag{3.9}$$

where B_α is a function which essentially is made out of λ and $\partial_\alpha \lambda$. Inserting the above in (3.8), the second term in (3.9) which is a Moyal bracket vanishes upon integrating over x, y. Then the integration over t can be easily performed and we find

$$\delta_g S = \frac{1}{2\pi\nu} \epsilon^{ij} \frac{\delta}{\delta\theta^{ij}} \int dxdy \ \lambda \Big|_{t=-\infty}^{t=+\infty} \ . \tag{3.10}$$

To evaluate the x, y integration it is more convenient to use the operatorial language developed in [30]:

$$\int dxdy \longleftrightarrow \pi \epsilon_{ij} \theta^{ij} \ \text{“tr”},$$

where "tr" is the trace over the Hilbert space of corresponding harmonic oscillator basis and any function λ can be expanded in terms of harmonic oscillator states

$$\lambda(t) = \sum a_{mn}(t) \mid n\rangle\langle m \mid \ .$$

The boundary conditions (3.4) is satisfied if

$$a_{mn}(t \to \infty) = \delta_{m,n}\ 2\pi K_n\ , \qquad K_n \in \mathbb{Z}\ . \tag{3.11}$$

Now, we have all the ingredients, inserting these all in (3.8)

$$\begin{aligned} \delta_g S &= \frac{1}{2\pi\nu}\epsilon^{ij}\ \pi\epsilon_{ij}\ 2\pi K \\ &= \frac{2\pi K}{\nu}\ , \qquad K \in \mathbb{Z}\ . \end{aligned} \tag{3.12}$$

However, in order to have a well-defined quantum theory $\delta_g S$ should be an integer multiple of 2π, i.e.

$$\frac{1}{\nu} \in \mathbb{Z}\ . \tag{3.13}$$

So, we see that in the noncommutative case, the level of the Chern-Simons should be quantized.

The NCU(N) winding number

All the above steps, before (3.9) work all the same for the NC$U(N)$ case. However, the (3.9) can be replaced with another identity:

$$\mathrm{Tr}(g^{-1} \star \partial_\alpha g) = i\mathrm{Tr}\partial_\alpha\lambda + \{B^a_\alpha, \lambda^a\}\ , \tag{3.14}$$

where ^{a} stands for the $U(N)$ group indices. Hence, again upon plugging back (3.14) in the (3.8), we face the level quantization.

So, we have shown that the winding number for the noncommutative plane case is non-zero and leads to level quantization. This result have also been observed using another calculational method [31]. We note that our result and also our arguments heavily rely on non-zero θ, furthermore the limit $\theta \to 0$ is not a smooth one. In particular for the NC$U(1)$ case again we need the level quantization.

It would be very interesting if we can generalize the general result of commutative case, namely $\Pi_3(G) = \mathbb{Z}$ for any compact group G, to the noncommutative case in more mathematical language [32].

For the noncommutative torus case of course, we should note that, since again there are some gauge transformations that are not smoothly connected to $\mathbf{1}$, and as it has been

discussed in [15], the winding number will still be an integer and hence in the torus case again we require the level quantization condition. We will discuss the torus case later in the next section.

4 Discussions and remarks

In this work we have studied some aspects of noncommutative Chern-Simons theories. First we discussed and reviewed the perturbative loop calculations. We discussed that we do not face the IR/UV mixing (the non-planar diagrams vanish), if we start with the noncommutative Yang-Mills-Chern-Simons and then after performing the loop calculations send the gauge coupling to infinity to obtain the pure noncommutative Chern-Simons action. We should recall that this limit is well-defined because (similar to the commutative case) we expect the quantum corrections on the Chern-Simons coupling $\frac{1}{\nu}$ to appear only at one loop level, and the one loop result (2.3) remain to all higher order loops. However, starting from the pure Chern-Simons we still find the non-planar IR divergences [17]. The (2.3) is remarkable remembering the statistics of the noncommutative Chern-Simons particles (1.5) [21]. Altogether it seems that for the NC$U(1)$ case, values of $\frac{1}{\nu}$ differing by an even integer are physically equivalent. Studying the behaviour of the noncommutative Chern-Simons under finite gauge transformations we have proved its invariance up to some integer multiple of the Chern-Simons coupling, and hence in order the noncommutative Chern-Simons theory to make sense at quantum level the coupling (level of Chern-Simons theory) should be quantized. We would like to stress that our proof and hence our result for NC$U(1)$ case does not hold in the $\theta^{\mu\nu}=0$ limit, i.e. the $\theta\to 0$ is not a smooth limit.

Here we have mainly discussed the Chern-Simons model on noncommutative plane. For the rational noncommutative torus case because there is a finite dimensional representation for the coordinates and generators of translations [23, 33], namely for $\Theta=\frac{P}{Q}$ we have a $Q\times Q$ representation, the arguments should be revised. However, we can still make some statements about the rational noncommutative torus case. Using the Morita equivalence, one can map $NCU(1)$ Chern-Simons into a $U(Q)$ Chern-Simons theory on a usual commutative torus but with a non-zero magnetic flux on the torus [24], i.e. we should add some terms linear in the gauge field to the Chern-Simons action. In other words, the classical equations of motion will not be the flat connections anymore. Instead we have to deal with the connections of constant curvature. The moduli space of these connections have also been studied well and we expect that it is possible to work out the precise form of the partition function for this

case. We postpone this to future work [32].

In the noncommutative case there is a possible extension for the Chern-Simons theory. In general since we have another two form in our problem, namely $\theta^{-1}_{\mu\nu} = \omega_{\mu\nu}$, we can add the term like

$$\int \mathrm{Tr}\left(\epsilon^{\mu\nu\alpha}\omega_{\mu\nu}A_\alpha\right)$$

to the Chern-Simons action, where $\omega_{\mu\nu}$ defines a constant 2-form which in a sense measures the noncommutativity of the space. We note that in 3 dimensions $\theta^{\mu\nu}$ has a zero eigenvalue, and so by $\theta^{-1}_{\mu\nu}$, we mean the inverse of θ in the noncommutative 2×2 block. So the extended action reads as

$$S = \frac{1}{4\pi\nu}\int \epsilon^{\mu\nu\alpha}\mathrm{Tr}\left(A_\mu \star \partial_\nu A_\alpha + \frac{2i}{3}A_\mu \star A_\nu \star A_\alpha\right) + \frac{i\gamma}{4\pi}\int \mathrm{Tr}\left(\epsilon^{\mu\nu\alpha}\omega_{\mu\nu}A_\alpha\right) , \qquad (4.1)$$

where γ is a dimensionless parameter, which is related to the unit of monopole charge. We also note that the gauge invariance of this action now implies that $\frac{1}{\nu} + \gamma \in \mathbb{Z}$. The possibility of having this additional term (which has no analog in the commutative case) has the significant effect of changing the equations of motion for the connection from the zero curvature to a constant curvature (proportional to ω).

We also discussed that the noncommutative Chern-Simons theory can be treated as a topological field theory. Then the next step to explore these theories is to find their partition function, and also the Wilson loops. It would also be interesting to consider the theory on the other noncommutative manifolds, such as noncommutative sphere$\times\mathbb{R}$ or $\Sigma\times\mathbb{R}$, with Σ being a noncommutative surface of arbitrary genus. However, our discussions in this work depend crucially on the definition of the $\star$-product, (1.3), and the fact that θ is a constant.

If one can work out the partition function for noncommutative Chern-Simons theories, besides the mathematical interests, this can be very interesting physically in the context of quantum Hall effect [21].

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References

[1] G. V. Dunne, *Aspects of Chern-Simons Theory*, hep-th/9902115.

[2] S. Grivin, *The Quantum Hall Effect: Novel Excitations and Broken Symmetries*, cond-mat/9907002.

[3] E. Witten, *Quantum Field Theory and Jones Polynomial*, *Comm. Math. Phys.* **121** (1989) 351.

[4] R. Dijkgraaf, E. Witten, *Topological Gauge Theories and Group Cohomology*, *Comm. Math. Phys.* **129** (1990) 393.

[5] S. Deser, R. Jackiw and S. Tempelton, *Three Dimensional Massive gauge theories*, *Phys. Rev. Lett.* **48** (1983) 975; *Topologically Massive gauge theories*, *Ann. Phys. NY* **140** (1982) 372.

[6] M. Blau, G. Thompson, *Do Metric Independent Classical Actions Lead to Topological Field Theories?*, *Phys. Lett.* **B255** (1991) 535.

[7] D. Birmingham, M. Blau, M. Rakowski, G. Thompson, *Topological Field Theories*, *Phys. Rep.* **209** (1991) 129.

[8] M. Asorey, F. Falceto, J.L. Lopez and G. Luzon, *Renormalization Ambiguities in Chern-Simons Theory*, *Phys. Rev. D.* **49** (1994) 5377;

G. Giavarini, C. P. Martin and F. Ruiz Ruiz, *Shift Versus No Shift in Local Regularizations of Chern-Simons Theory*, *Phys. Lett.* **B332** (1994) 345.

[9] R. Pisarski, S. Rao, *Topologically Massive Chromodynamics in the Perturbative Regime*, *Phys. Rev.* **D32** (1985) 2081.

[10] W. Chen, G. Semenoff and Y-S. Wu, *Finite Renormalization of Chern-Simons Gauge Theory*, *Physics, Geometry and Topology*, *Proccedings of Banff NATO Advanced Studies Institute*, 1989, H-C. Lee (Ed.) (Plenum, 1990); *Phys. Rev. D.* **44** (1991) R1625; *Phys. Rev. D.* **46** (1992) 5521.

G. Semenoff, P. Sodano and Y-S. Wu, *Phys. Rev. Lett.* **62** (1989) 715.

[11] F. T. Brandt, Ashok Das, J. Frenkel, *Absence of Higher Order Corrections to The Non-Abelian Chern-Simons Coefficient*, *Phys. Lett.* **B494** (2000) 339, hep-th/0009236;

Absence of Higher Order Corrections to The Non-Abelian Topological Mass Term, hep-th/0012087.

[12] J. Lukierski, P. C. Stichel, W. J. Zakrzewski, *Galilean-Invariant (2+1)-Dimensional Models with a Chern-Simons-Like Term and D=2 Noncommutative Geometry*, *Annals Phys.* **260** (1997) 224, hep-th/9612017.

[13] A. Connes, M. R. Douglas, A. Schwarz, *Noncommutative Geometry and Matrix Theory: Compactification on Tori*, *JHEP* **9802** (1998) 003, hep-th/9711162.

N. Seiberg, E. Witten, "String Theory and Noncommutative Geometry", *JHEP* **09** (1999) 032, hep-th/9908142.

[14] A. Micu, M.M. Sheikh-Jabbari, *Noncommutative* Φ^4 *Theory at Two Loops*, *JHEP* **01** (2001) 025, hep-th/0008057.

[15] T. Krajewski, *Gauge Invariance of the Chern-Simons Action in Noncommutative Geometry*, math-ph/9810015.

[16] C-S. Chu, *Induced Chern-Simons and WZW action in Noncommutative Spacetime*, *Nucl. Phys.* **B580** (2000) 352, hep-th/0003007.

[17] A.A. Bichl, J. M. Grimstrup, V. Putz and M. Schweda, *Perturbative Chern-Simons Theory on Noncommutaitve* $\mathbb{R}^3$, *JHEP* **0007** (2000) 046, hep-th/0004071.

[18] G-H. Chen, Y-S. Wu, *One Loop Shift in Noncommutaitve Chern-Simons Coupling*, *Nucl. Phys.* **B593** (2001) 562, hep-th/0006114.

[19] N. Grandi and G. A. Silva, *Chern-Simons Action in Noncommutative Space*, hep-th/0010113.

A. P. Polychronakos, *Noncommutative Chern-Simons Terms and the Noncommutative Vacuum*, *JHEP* **0011** (2000) 008, hep-th/0010264.

[20] G.S. Lozano, E.F. Moreno, F.A. Schaposnik, *Self-Dual Chern-Simons Solitons in Noncommutaitve Space*, hep-th/0012266.

A. Khare, M. B. Paranjape, *Solitons in 2+1 Dimensional Noncommutative Maxwell-Chern-Simons Higgs Theories*, hep-th/0102016.

[21] L. Susskind, *The Quantum Hall Fluid and Noncommutative Chern-Simons Theory*, hep-th/0101029.

[22] H. Weyl, *The theory of groups and quantum mechanics* (Dover, New York, 1950).

H. Grosse, C. Klimčík and P. Prešnajder, *Int. J. Theor. Phys.* **35** (1996) 231 , *Comm. Math. Phys.* **178** (1996) 507 and **185** (1997) 155.

[23] M. Chaichian, A. Demichev, P. Presnajder, M. M. Sheikh-Jabbari, A. Tureanu, *Quantum Theories on Noncommutative Spaces with Nontrivial Topology: Aharonov-Bohm and Casimir Effects*, hep-th/0101209.

[24] A. Connes, *Noncommutative Geometry* (Academic Press, New York, 1994).

C. Hofman, E. Verlinde, *U-Duality of Born-Infeld on the Noncommutative Two-Torus*, *JHEP* **9812** (1998) 010, hep-th/9810116.

[25] S. Minwalla, M. Van Raamsdonk, N. Seiberg, *Noncommutative Perturbative Dynamics*, hep-th/9912072.

[26] A. Matusis, L. Susskind, N. Toumbas, *The IR/UV Connection in the Noncommutative Gauge Theories*, hep-th/0002075.

[27] M.M. Sheikh-Jabbari, *One Loop Renormalizability of Supersymmetric Yang-Mills Theories on Noncommutative Two-Torus*, *JHEP* **9906** (1999) 015, hep-th/9903107.

A. Armoni, *Comments on Perturbative Dynamics of Noncommutative Yang-Mills Theory*, *Nucl. Phys.* **B593** (2001) 229, hep-th/0005208.

[28] A. Das, M.M. Sheikh-Jabbari, *Absence of Higher Order Corrections to Noncommutative Chern-Simons Coupling*, hep-th/0103139.

[29] M.A. Rieffel, *Noncommutative Tori -A Case Study of Noncommutative Differentiable Manifolds*, Contemp. Math. **105**, (1991) 191; *Deformation Quantization for Actions of* $\mathbb{R}^d$, 51 **02** AMS Mem. no.506, 1993.

A. Connes, M. Rieffel, *Yang-Mills for Noncommutaitve Two-Tori*, Contemp. Math. Oper. Algebra. Math. Phys. **62**, AMS (1987).

[30] J. A. Harvey, P. Kraus, F. Larsen, *Exact Noncommutative Solitons*, *JHEP* **0012** (2000) 024, hep-th/0010060.

J. A.Harvey, *Komaba Lectures on Noncommutative Solitons and D-Branes*, hep-th/0102076.

[31] V.P. Nair, A.P. Polychronakos, *On Level Quantization for the Noncommutative Chern-Simons Theory*, hep-th/0102181.

[32] T. Krajewski, M.M. Sheikh-Jabbari, *Work in Progress*.

[33] J. Ambjorn, Y.M. Makeenko, J.Nishimura and R.J.Szabo, *Lattice Gauge Fields and Discrete Noncommutative Yang-Mills Theory*, *JHEP* **0005** (2000) 023, hep-th/0004147.